天 氣 學 二 (Synoptic Meteorology II) 上課時間: 10:20~12:10 Wednesday, B105 授課教師: 游政谷 email: yuku@ntu.edu.tw

Chapter 2 Quasi-geostrophic Theory and Its Application

2.2 Application of QG Tendency Equation

$$\begin{pmatrix} \nabla^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \end{pmatrix} \mathcal{X} = -f_0 \overrightarrow{V_g} \cdot \nabla \left(\frac{1}{f_0} \nabla^2 \Phi + f \right) + \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \left[-\overrightarrow{V_g} \cdot \nabla \left(\frac{\partial \Phi}{\partial p} \right) \right]$$

$$(A) \qquad (B) \qquad (C)$$

Term A: Proportional to $-\mathcal{X}$ Term B: Geostrophic absolute vorticity advection Term C: Differential thickness advection Schematic illustrating the contributions of the geostrophic absolute vorticity advection to height falls and rises (note: the height tendency is zero at the vorticity maximum)



Schematic illustrating (a) a digging trough and (b) a lifting trough; these troughs are associated with jets and can produce a net height tendency near the base of the trough



Idealized schematics of (a) a digging trough and (b) a lifting trough. Geopotential (solid contours), select wind vectors (arrows), and the locations of vorticity and vorticity advection maxima are labeled (from Bluestein 1993).

1200 UTC 10 Jan 2010







1200 UTC 11 Jan 2010 Ws



Summary of 500-mb characteristics for a lifting trough on 10–11 Jan 2010: (a) 500-mb geopotential height analysis valid 1200 UTC 10 Jan contour interval 6 dam; (c) as in (a) but including 500-mb isotachs (kt, shaded as in legend); (e) as in (a) but including 500-mb absolute vorticity (every 2×10^{-5} s⁻¹, shaded as in legend); and (b),(d),(f) as in (a)–(c), but valid 1200 UTC 11 Jan 2010. Graphics are based on analyses from the GFS model.

(b)

(d)

An example showing the evolution of a lifting trough



Summary of 500-mb characteristics for a digging trough on 22–23 Oct 2008: (a),(c),(e) as in Fig. 2.14, but valid 0000 UTC 22 Oct; (b),(d),(f) as in (a),(c),(e) but valid 0000 UTC 23 Oct.

An example showing the evolution of a digging trough

Schematics illustrating how local maximum of warm and cold advection contribute to changes in height fields



Idealized schematics depicting geopotential height changes associated with the differential thermal advection term in the QG height tendency equation (2.32).

Contribution of cold advection to amplification of the upper-level trough through the QG forcing



Cold advection and an amplifying upper trough: (a) 0-h GFS forecast valid 1200 UTC 26 Oct 2009; (b) 18-h forecast valid 0600 UTC 27 Oct 2009; (c) 42-h forecast valid 0600 UTC 28 Oct 2009. (d) North-south cross section of temperature advection (shaded every 10×10^{-5} K s⁻¹ as in legend at bottom of panel) based on GFS analysis valid 12 UTC 27 Oct 2009.

2.3 Application of QG Omega Equation

$$\begin{pmatrix} \nabla^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \end{pmatrix} \omega = \frac{f_0}{\sigma} \frac{\partial}{\partial p} \left[\overrightarrow{V_g} \cdot \nabla \left(\frac{1}{f_0} \nabla^2 \Phi + f \right) \right] + \frac{1}{\sigma} \nabla^2 \left[\overrightarrow{V_g} \cdot \nabla \left(- \frac{\partial \Phi}{\partial p} \right) \right]$$

$$(A) \qquad \qquad (B) \qquad \qquad (C)$$

Term A: Proportional to -ω Term B: Differential vorticity advection Term C: Thermal advection Note: In real atmospheric, Orographic or frictional effects, not included in QG, likely overpower the QG signal. Moreover, rising air does not guarantee precipitation or even clouds

Additional considerations when using QG Omega Eq. in practical forecasting:

- How intense is the forcing for ascent?
- Do the two right-hand terms reinforce or cancel?
- Is there sufficient moisture available?
- Are non-QG mechanisms at work that could obscure the QG signal?

Schematics illustrating the physical cause for the QG-forced vertical motions; thermal wind balance is present at the initial time but it is being disrupted due to advection of zonal geostrophic winds







Figure 2.4. Vertical cross section B–C with orientation as indicated in Fig. 2.3. Dashed colored arrows indicate sense of ageostrophic circulation. Black dashed line represents a geopotential height contour, with exaggerated vertical displacement.

Descent and ascent forced by the differential vorticity advection in the QG Omega Equation



Figure 2.5. As in Fig. 2.3, but showing idealized geostrophic relative vorticity distribution in shaded colors, along with areas of significant differential vorticity advection and forcing for ascent and descent.

Distribution of vorticity advection associated a jet streak



Figure 2.6. Idealized jet streak at the 500-mb level. Solid black lines are geopotential height contours, dashed red and blue lines represent relative vorticity isopleths, and the designations CVA and AVA indicate regions of cyclonic and anticyclonic vorticity advection, respectively.

A real example of vertical motions forced by differential vorticity advection





Figure 2.7. Analysis from the Rapid Update Cycle (RUC) model valid 0600 UTC 10 Sep 2008 for the U.S. Pacific Northwest: (a) 500-mb height (solid contours, interval is 6 dam), wind barbs, and absolute vorticity (shaded every $2 \times 10^{-5} \text{ s}^{-1}$ beginning at $8 \times 10^{-5} \text{ s}^{-1}$); (b) as in (a) except with 500-mb vorticity advection replacing vorticity (contoured every $12 \times 10^{-10} \text{ s}^{-2}$); (c) as in (a) except with omega (shaded as in legend, interval is $2 \times 10^{-1} \text{ Pa s}^{-1}$) replacing vorticity.



Thermal advection in the Omega equation





Figure 2.8. NAM forecasts for an East Coast storm: (a) 54-h forecast valid 1800 UTC 10 Sep 2009: 500-mb height (blue contours, interval is 20 m), sea level pressure (red contours, interval is 3 mb); (b) as in (a) but including 850-mb temperature advection [shaded, interval is 6×10^{-5} K s⁻¹, warm (cool) colors indicate warm (cold) advection]; (c) 54-h model forecast sounding, skew *T*-log*p* format, for Philadelphia, PA.

An example showing possible cancellation between the two right-hand terms of the Omega equation (note: wind backing to the east of the trough)



Figure 2.9. GFS forecast of 500-mb height and sea level pressure, valid 1800 UTC 13 Sep 2008: 500-mb height (solid blue lines, interval is 6 dam) and sea level pressure (dashed red contours, interval is 2 mb).

The Q-vector form of the QG Omega equation can be written as

$$\left(\nabla^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2}\right)\omega = -2\nabla \cdot \vec{Q}$$

where

$$\vec{Q} = -\frac{R}{\sigma p} \begin{bmatrix} \frac{\partial \overrightarrow{V_g}}{\partial x} & \cdot & \nabla \theta \\ \frac{\partial \overrightarrow{V_g}}{\partial y} & \cdot & \nabla \theta \end{bmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = -\frac{R}{\sigma p} \begin{bmatrix} \frac{\partial u_g}{\partial x} \frac{\partial \theta}{\partial x} & + & \frac{\partial v_g}{\partial x} \frac{\partial \theta}{\partial y} \\ \frac{\partial u_g}{\partial y} \frac{\partial \theta}{\partial x} & + & \frac{\partial v_g}{\partial y} \frac{\partial \theta}{\partial y} \end{bmatrix}.$$

Forcing for ascent (descent) when Q vectors converge (diverge)

Q-vector analysis in this case suggests a stronger forcing of the differential vorticity advection compared to thermal advection



Figure 2.11. GFS 78-h forecast valid 1800 UTC 13 Sep 2008 (as in Fig. 2.9): 500-mb height (solid black lines, interval is 6 dam), **Q** vectors (arrows), and **Q**-vector convergence and divergence (dashed blue for convergence, solid red for divergence).