天 氣 學 二
(Synoptic Meteorology II)

上課時間: 10:20～12:10 Wednesday, B105

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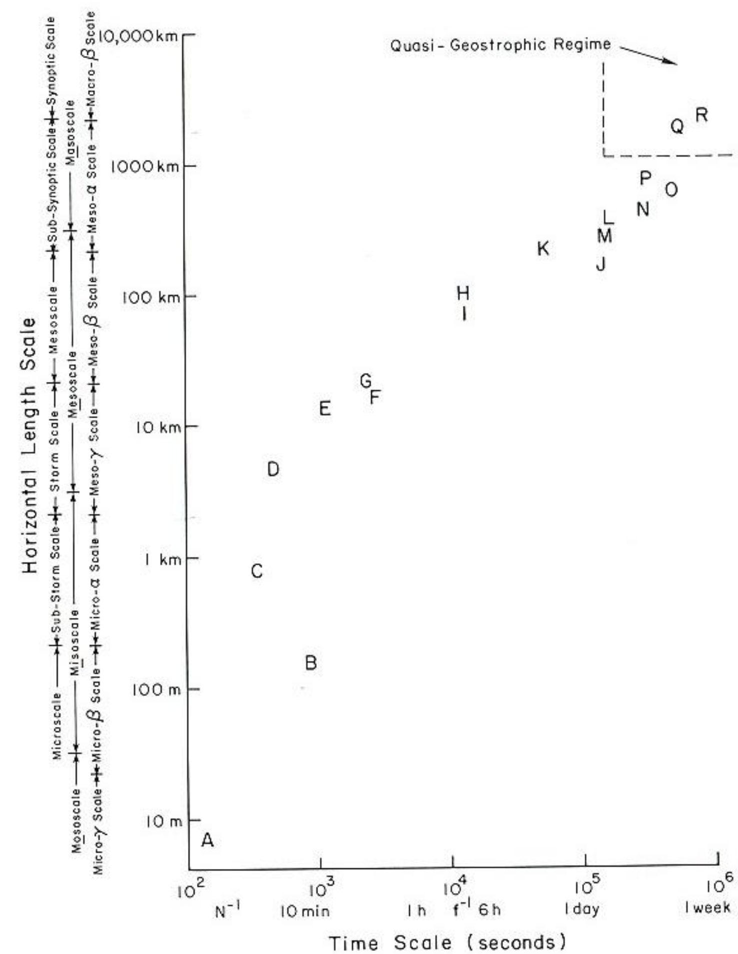
Chapter 2 Quasi-geostrophic Theory and Its Application

2.1 QG近似與方程組

Q: Extratropical cyclones and anticyclones

R: Troughs and ridges in the baroclinic westerlies

Only Q and R fit well in the quasi-geostrophic regime



Horizontal-length scales and time scales for the following atmospheric phenomena: A, dust devils; B, tornadoes and waterspouts; C, cumulus clouds; D, downbursts; E, gust fronts; F, mesocyclones; G, thunderstorms; H, sea/land/lake breezes, mountain–valley circulations, and meso-highs and meso-lows; I, precipitation bands; J, coastal fronts; K, mesoscale convective systems; L, the low-level jet; M, the dryline; N, "bombs" and tropical cyclones; O, upper-level jets; P, surface fronts; Q, extratropical cyclones and anticyclones; and R, troughs and ridges in the baroclinic westerlies.

等壓座標水平動量方程：

$$\frac{du}{dt} - fv = -\frac{\partial \Phi}{\partial x} \quad (1)$$

$$\frac{dv}{dt} + fu = -\frac{\partial \Phi}{\partial y} \quad (2)$$

其中$\frac{d}{dt} \equiv \left(\frac{\partial}{\partial t}\right)_p + u\left(\frac{\partial}{\partial x}\right)_p + v\left(\frac{\partial}{\partial y}\right)_p + \omega\frac{\partial}{\partial p} \quad (3)$

靜力方程：

$$\frac{\partial \Phi}{\partial p} = -\alpha = -\frac{RT}{P} \quad (4)$$

連續方程：

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \quad (5)$$

熱力能量方程：

$$\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)_p - S_p\omega = \frac{\dot{q}}{c_p} \quad (6) \qquad S_p \equiv -T\frac{\partial \ln \theta}{\partial P}$$

以上方程雖然已相當簡化，要想藉此瞭解綜觀級系統的結構仍嫌十分棘手，所以設法利用幅度分析導出一組遠為簡化的方程來，以便針對綜觀斜壓系統作適當的診斷與預測。

由(4)知 $\mathrm{T}=-\frac{P}{R}\frac{\partial\Phi}{\partial p}$ 代入(6)

$$\Rightarrow\left[\frac{\partial}{\partial t}\left(-\frac{P}{R}\frac{\partial\Phi}{\partial p}\right)+u\frac{\partial}{\partial x}\left(-\frac{P}{R}\frac{\partial\Phi}{\partial p}\right)+v\frac{\partial}{\partial y}\left(-\frac{P}{R}\frac{\partial\Phi}{\partial p}\right)\right]_p-S_p\omega=\frac{\dot{q}}{C_p}$$

同乘$\frac{R}{P}\Rightarrow\left[\frac{\partial}{\partial t}\left(-\frac{\partial\Phi}{\partial p}\right)+u\frac{\partial}{\partial x}\left(-\frac{\partial\Phi}{\partial p}\right)+v\frac{\partial}{\partial y}\left(-\frac{\partial\Phi}{\partial p}\right)\right]_p-\frac{RS_p}{P}\omega=\frac{R}{P}\frac{\dot{q}}{C_p}$ (7)

定義靜力穩定參數$\sigma\equiv-\frac{\alpha}{\theta}\frac{\partial\theta}{\partial p}=\frac{RS_p}{P}$，$\mathrm{P}\alpha=RT$

上式$\Rightarrow\frac{\partial}{\partial t}\left(-\frac{\partial\Phi}{\partial p}\right)+u\frac{\partial}{\partial x}\left(-\frac{\partial\Phi}{\partial p}\right)+v\frac{\partial}{\partial y}\left(-\frac{\partial\Phi}{\partial p}\right)-\sigma\omega=\frac{R}{P}\frac{\dot{q}}{C_p}$

當為靜力穩定之大氣$\sigma>0\iff\frac{\partial\theta}{\partial p}<0$

對綜觀尺度系統而言，水平風場近似於地轉風速，

因此我們定義 $\vec{V}=u\hat{i}+v\hat{j}\equiv\overrightarrow{V_g}=\frac{\hat{k}\times\nabla\Phi}{f}$ 代入上式

另外，假設非絕熱貢獻$\dot{q}$比其他項小(此項並非完全不重要，有時仍有機會扮演重要角色，例如，當天氣系統伴隨明顯降雨時或考量較長時間尺度時)

$$\Longrightarrow \frac{\partial}{\partial t}\left(-\frac{\partial\Phi}{\partial p}\right)=-\overrightarrow{V_g}\cdot\nabla\left(-\frac{\partial\Phi}{\partial p}\right)+\sigma\omega \quad (8)$$

(8) 式為靜力平衡下之熱力能量方程，只有二個變數Φ及ω

我們對(8)式之每項物理意義加以說明：

$-\frac{\partial\Phi}{\partial p}$可視之為溫度，因$\frac{\partial\Phi}{\partial p}=-\alpha=-\frac{RT}{P}$

在靜力平衡之大氣下，$-\frac{\partial\Phi}{\partial p}$是和兩個等壓面間的平均溫度T成正比，

當然$-\frac{\partial\Phi}{\partial p}$也可視為厚度，$\because -\frac{\partial\Phi}{\partial p}\approx-\frac{\delta\Phi}{\delta p}$ (只不過取$\delta p\longrightarrow 0$之極限觀念而已)，

所以$\frac{\partial}{\partial t}\left(-\frac{\partial\Phi}{\partial p}\right)$乃和溫度隨時間之局部變率成正比，

而$-\overrightarrow{V_g}\cdot\nabla\left(-\frac{\partial\Phi}{\partial p}\right)$表示以地轉風計算之溫度平流，

最後一項$\sigma\omega$ 通常稱之為絕熱冷卻(增溫)項，指空氣塊在靜力穩定環境中上升膨脹冷卻(下沈壓縮增溫)所致絕熱溫度變化。

接下來，將水平動量方程以渦度方程代替

$$(1)、(2) \Longrightarrow \underset{(A)}{\frac{\partial\zeta}{\partial t}} = \underset{(B)}{-\vec{V}\cdot\nabla(\zeta+f)} \underset{(C)}{-\omega\frac{\partial\zeta}{\partial p}} \underset{(D)}{-(\zeta+f)\nabla\cdot\vec{V}} + \underset{(E)}{\left(\frac{\partial u}{\partial p}\frac{\partial\omega}{\partial y} - \frac{\partial v}{\partial p}\frac{\partial\omega}{\partial x}\right)} \quad \text{(等壓座標)}$$

(A) 相對渦度的局部變率
(B) 絕對渦度之水平平流
(C) 相對渦度的垂直平流
(D) 輻散項或拉伸項
(E) 扭轉或傾側項

利用尺度分析，我們從中緯度綜觀級尺度可將渦度方程作一簡化

1. 忽略垂直平流項(C)和扭轉項(E)
2. 在(D)輻散項中忽略$\zeta \Longrightarrow -f\nabla\cdot\vec{V}$ $(\zeta \ll f)$
3. 在(B) $\vec{V}$以$\overrightarrow{V_g}$代替
4. 相對渦度ζ以ζ_g代替，$\zeta_g \equiv \dfrac{\partial v_g}{\partial x} - \dfrac{\partial u_g}{\partial y}$

此外，另有一種簡化是用常數f_0代替隨緯度變化的$\mathrm{f} = 2\Omega\sin\phi = f(\phi)$，

f用泰勒級數展開$f = f_0 + \beta y +$其他高次項，其中$\beta \equiv \left(\dfrac{df}{dy}\right)_{\phi_0}$，$\phi_0$處之$y = 0$

$\Longrightarrow$前二項之數量級比$\dfrac{\beta L}{f_0} \sim \dfrac{\cos\phi_0 L}{\sin\phi_0 a}$

故當運動之緯度範圍遠比地球半徑為小時$\left(\dfrac{L}{a} \ll 1\right)$則$f \approx f_0$

當然在平流項(B)中不能設$f \approx f_0$，不過可假設$\dfrac{df}{dy} = \beta =$ 常數，此種簡化稱β平面估擬。

經過上述簡化後$\Rightarrow \frac{\partial \zeta_g}{\partial t} = -\overrightarrow{V_g} \cdot \nabla(\zeta_g + f) - f_0 \nabla \cdot \vec{V}$稱為準地轉渦度方程

注意：$\zeta_g = \frac{\nabla^2 \Phi}{f_0}$ $\overrightarrow{V_g} = \frac{\hat{k} \times \nabla \Phi}{f_0}$ ，兩者計算都用固定的柯氏參數f_0

注意：輻散項之$\vec{V}$沒有以$\overrightarrow{V_g}$代替，乃因$\nabla_p \cdot \overrightarrow{V_g} = 0$，當認為$f = f_0 =$常數時

由(5)知 $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial \omega}{\partial p} \Rightarrow \nabla \cdot \vec{V} = -\frac{\partial \omega}{\partial p}$ 代入上式，

因此轉地轉渦度方程有另外一種形式

$$\Rightarrow \underbrace{\frac{\partial \zeta_g}{\partial t}}_{(A)} = \underbrace{-\overrightarrow{V_g} \cdot \nabla(\zeta_g + f)}_{(B)} + f_0 \frac{\partial \omega}{\partial p} \qquad (9)$$

此式可用來診斷說明ω場，因為(A)、(B)兩者都能相當準確地予以估擬，所以由上式計算出來的ω估擬值通常比直接用連續方程計算結果為佳。

(8)、(9)式只含兩個變數Φ及ω，故(8)、(9)兩式構成一個以Φ及ω為變數的封閉性預報方程組。所以運用(8)、(9)二式而獲得之新式子，使我們有辦法單靠某一瞬時之實測Φ場便可$\Rightarrow \frac{\partial \Phi}{\partial t}$及$\omega$場。因此不必直接觀測速度場就能預測中緯度綜觀氣流運動的演變。 (8)、(9)合稱為準地轉系統(quasi-geostrophic system)，是綜觀動力氣象學之核心。

趨勢方程(Tendency equation)：

定義：$\mathcal{X} \equiv \dfrac{\partial \Phi}{\partial t}$

$$\Longrightarrow \quad (8) \Longrightarrow \frac{\partial \mathcal{X}}{\partial p} = -\overrightarrow{V_g} \cdot \nabla \left(\frac{\partial \Phi}{\partial p}\right) - \sigma\omega \quad (12)$$

$$(9) \Longrightarrow \nabla^2 \mathcal{X} = -f_0 \overrightarrow{V_g} \cdot \nabla \left(\frac{1}{f_0} \nabla^2 \Phi + f\right) + {f_0}^2 \frac{\partial \omega}{\partial p} \quad (13)$$

$$\frac{\partial}{\partial p}(12) + \frac{\sigma}{{f_0}^2}(13)$$

$$\Longrightarrow \underbrace{\left(\nabla^2 + \frac{{f_0}^2}{\sigma} \frac{\partial^2}{\partial p^2}\right)\mathcal{X}}_{(A)} = \underbrace{-f_0 \overrightarrow{V_g} \cdot \nabla \left(\frac{1}{f_0} \nabla^2 \Phi + f\right)}_{(B)} + \underbrace{\frac{{f_0}^2}{\sigma} \frac{\partial}{\partial p}\left[-\overrightarrow{V_g} \cdot \nabla \left(\frac{\partial \Phi}{\partial p}\right)\right]}_{(C)} \quad (\sigma\text{假定為常數})$$

……重力位趨勢方程(Geopotential or Height Tendency Equation)

Omega方程：

由(12)、(13)式可消去$\mathcal{X}$ $\Longrightarrow$ ω方程如下：

$$\underbrace{\left(\nabla^2+\frac{{f_0}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{(A)}=\underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\overrightarrow{V_g}\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi+f\right)\right]}_{(B)}+\underbrace{\frac{1}{\sigma}\nabla^2\left[\overrightarrow{V_g}\cdot\nabla\left(-\frac{\partial\Phi}{\partial p}\right)\right]}_{(C)}$$

上式內僅含空間導數，所以這是用瞬時之Φ場來對ω場之診斷方程。

ω方程不像用連續方程求ω值，非得有精確的水平風觀測值不可，而只要某定時之Φ的觀測就可以。